Inductance

CHAPTER 36

36–1 Inductance

If two coils are near each other, a current i in one coil will set up a flux Φ_B through the second coil. If this flux is changed by changing the current, an induced emf will appear in the second coil according to Faraday's law. However, two coils are not needed to show an inductive effect. An induced emf appears in a coil if the current *in that same coil* is changed. This is called *self-induction* and the electromotive force produced is called a *self-induced emf*. It obeys Faraday's law of induction just as other induced emfs do.

Consider first a "close-packed" coil, a toroid, or the central section of a long solenoid. In all three cases the flux Φ_B set up in each turn by a current i is essentially the same for every turn. Faraday's law for such coils (Eq. 35–2)

$$\mathcal{E} = -\frac{d(N\Phi_B)}{dt} \tag{36-1}$$

shows that the *number of flux linkages* $N\Phi_B$ (N being the number of turns) is the important characteristic quantity for induction. For a given coil, provided no magnetic materials such as iron are nearby, this quantity is proportional to the current i, or

$$\boxed{N\Phi_B = Li,} \tag{36-2}$$

in which L, the proportionality constant, is called the *inductance* of the device.

From Faraday's law (see Eq. 36-1) the induced emf can be written as

$$\boxed{\mathcal{E} = -\frac{d(N\Phi_B)}{dt} = -L\frac{di}{dt}} \tag{36-3a}$$

Written in the form

$$\boxed{L = -\frac{\mathcal{E}}{di/dt},} \qquad (36\text{–}3b)$$

this relation may be taken as the defining equation for inductance for coils of all shapes and sizes, whether or not they are close-packed and whether or not iron or other magnetic material is nearby. It is analogous to the defining relation for capacitance, namely

$$C = \frac{q}{V}.$$

If no iron or similar materials are nearby, L depends only on the geometry of the device. In an *inductor* (symbol) the presence of a *magnetic field* is the significant feature, corresponding to the presence of an *electric field* in a *capacitor*.

The unit of inductance, from Eq. 36–3b, is the volt-sec/amp. A special name, the *henry*, has been given to this combination of units, or

$$\boxed{1 \text{ henry} = 1 \text{ volt-sec/amp.}}$$

The unit of inductance is named after Joseph Henry (1797–1878), an American physicist and a contemporary of Faraday. Henry independently discovered the law of induction at about the same time Faraday did. The units *millihenry* (1 mh = 10^{-3} henry) and *microhenry* (1 μh = 10^{-6} henry) are also commonly used.

The direction of a self-induced emf can be found from Lenz's law. Suppose that a steady current i, produced by a battery, exists in a coil. Let us suddenly reduce the (battery) emf in the circuit to zero. The current i will start to decrease at once; this *decrease* in current, in the language of Lenz's law, is the "change" which the self-induction must oppose. To oppose the falling current, the induced emf must point in the same direction as the current, as in Fig. 36–1a. When the current in a coil is increased, Lenz's law shows that the self-induced emf points in the *opposite* direction to that of the current, as in Fig. 36–1b. In each case the self-induced emf acts to op-

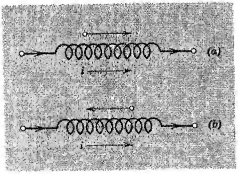

Fig. 36–1 In (a) the current i is *decreasing* and in (b) it is *increasing*. The self-induced emf $\mathcal{E}_L$ opposes the *change* in each case.

pose the *change* in the current. The minus sign in Eq. 36-3 shows that $\mathcal{E}$ and di/dt are opposite in sign, since L is always a positive quantity.

36-2 Calculation of Inductance

It has proved possible to make a direct calculation of capacitance in terms of geometrical factors for a few special cases, such as the parallel-plate capacitor. In the same way, it is possible to calculate the self-inductance L for a few special cases.

For a close-packed coil with no iron nearby, we have, from Eq. 36-2,

$$\boxed{L = \frac{N\Phi_B}{i}.} \tag{36-4}$$

Let us apply this equation to calculate L for a section of length l near the center of a long solenoid. The number of flux linkages in the length l of the solenoid is

$$N\Phi_B = (nl)(BA),$$

where n is the number of turns per unit length, B is the magnetic induction inside the solenoid, and A is the cross-sectional area. From Eq. 34-7, B is given by

$$B = \mu_0 ni$$

Combining these equations gives

$$N\Phi_B = \mu_0 n^2 liA.$$

Finally, the inductance, from Eq. 36-4, is

$$\boxed{L = \frac{N\Phi_B}{i} = \mu_0 n^2 lA.} \tag{36-5}$$

The inductance of a length l of a solenoid is proportional to its volume (lA) and to the square of the number of turns per unit length. Note that it depends on geometrical factors only. The proportionality to n^2 is expected. If the number of turns per unit length is doubled, not only is the *total* number of turns N doubled but also the flux *through each turn* Φ_B is also doubled, an over-all factor of four for the flux linkages $N\Phi_B$, hence also a factor of four for the inductance (Eq. 36-4).

▶ **Example 1.** Derive an expression for the inductance of a toroid of rectangular cross section as shown in Fig. 36-2. Evaluate for $N = 10^3$, $a = 5.0$ cm, $b = 10$ cm, and $h = 1.0$ cm.

The lines of **B** for the toroid are concentric circles. Applying Ampère's law,

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 i,$$

to a circular path of radius r yields

$$(B)(2\pi r) = \mu_0 i_0 N,$$

where N is the number of turns and i_0 is the current in the toroid windings; recall that